6 SEM TDC DSE MTH (CBCS) 1 (H)

2025

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

(Hydromechanics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* State two methods of studying fluid motion mathematically. 1

(b) The motion of a fluid is said to be _____ when the vorticity of every fluid particle is zero.

(Fill in the blank) 1

(c) Define velocity potential. Under what condition, the flow is known as potential kind? 3

(d) Determine the acceleration at the point (2, 1, 3) at $t = 0{\cdot}5$, if $u = yz + t$, $v = xz - t$ and $w = xy$. 3

(e) Deduce the equation of continuity by Euler's method. 7

Or

Show that a fluid of constant density can have a velocity with components

$$u = -\frac{2xyz}{(x^2+y^2)^2},\ v = \frac{(x^2-y^2)z}{(x^2+y^2)^2} \text{ and } w = \frac{y}{x^2+y^2}$$

Find the vorticity vector.

2. (a) Write the equation of motion of an incompressible fluid under impulsive force. 1

(b) Impulsive pressure at any point in a fluid is the same in every direction.
(State True or False) 1

(c) If the motion of an ideal fluid, for which density is a function of pressure p only, is steady and the external forces are conservative, then prove that there exists a family of surfaces which contains the stream lines and vortex lines. 5

(d) A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being Π. Show that the pressure at a distance r from the centre immediately falls to $\Pi(1-a/r)$. 5

Or

A stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d, if V and v be the corresponding velocities of the stream and if the motion be supposed to be that of the divergence from the vertex of the cone, then prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2-V^2)/2k}$$

where k is the pressure divided by the density and supposed constant.

3. (a) State Green's theorem. 1

(b) State whether True or False : 1

Acyclic irrotational motion is possible in a liquid bounded entirely by fixed rigid walls.

(c) State and prove Kelvin minimum energy theorem. 6

Or

(i) Deduce from Green's theorem that the total flow of liquid into any closed region at any instant is zero. 2

(ii) Show that in irrotational motion the velocity cannot be a maximum in the interior of the fluid. 4

4. *(a)* Define pressure and hydrostatics paradox. 2

(b) State whether True or False : 1

When two fluids of different densities are at rest under gravity and do not mix, their surface of separation is a horizontal plane.

(c) Prove that surfaces of equal pressure are intersected orthogonally by the lines of forces. 3

(d) Three liquids, whose densities are in AP, fill a semicircular tube whose bounding diameter is horizontal. Prove that the depth of one of the common surfaces is double that of the other. 5

(e) A tube in the form of a parabola held its vertex downwards and axis vertical is filled with two different liquids of

densities ρ and ρ'. If the distances of the free surfaces of the liquids from the focus are r and r' respectively, then show that the distance of the common surface from the focus is

$$\frac{r\rho - r'\rho'}{\rho - \rho'}$$ 6

Or

(i) Prove that the surfaces of equal pressure are intersected orthogonally by the line of forces. 3

(ii) If a fluid is at rest under the forces X, Y, Z per unit mass, then find the curves of equal pressure and equal density. 3

5. *(a)* Define force of buoyancy. 1

(b) A cone full of water is placed on its side on a horizontal table. Show that the thrust on the base is $3w\sin\alpha$, where w is the weight of the contained fluid and 2α is the vertical angle of the cone. 4

(c) A semicircular lamina is immersed in a liquid with the diameter in the surface. Find the depth of the centre of pressure. 6

Or

Find the CP of a parallelogram immersed in a homogeneous liquid with one side in the free surface.

(d) A hollow cone is placed with its vertex upward on a horizontal table and liquid is poured in through a small hole in the vertex. If the cone begins to rise when the weight of the liquid poured in it equals its weight, prove that its weight is to the weight of the liquid required to fill the cone is as $(9-3\sqrt{3}) : 4$. 5

Or

A solid circular cone of uniform material and height h and of vertical angle 2α, is made of uniform material and floats in water with its axis vertical and vertex downwards and a length h' of the axis is immersed. The cone is bisected by a vertical plane through the axis and the two parts are hinged together at the vertex. Show that the two parts will remain in contact if $h' > h\sin^2\alpha$.

6. *(a)* Fill in the blank : 1

If the _____ coincide with the centre of gravity, the equilibrium is neutral.

(b) Define surface of buoyancy. 1

(c) A cylinder floating with its axis horizontal and in the surface, is displaced in the vertical plane through the axis. Discuss its stability of equilibrium. 5

(d) A solid cone of semivertical angle α, specific gravity σ floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Determine the condition for which the equilibrium is stable. 5

Or

Prove that the tangent at any point of surface of buoyancy is parallel to the corresponding plane of floatation.

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