

6 SEM TDC DSE MTH (CBCS) 6 (H)

2025

(May)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-6

(Mathematical Methods)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Write the period of the function $\sin 2x$. 1

(b) Write the values of the coefficients a_n and b_n in the Fourier expansion of the function $f(x)$ in the interval $(-\pi, \pi)$ as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cosh x + b_n \sinh x) \quad 2$$

(c) Find a Fourier series for the function $f(x) = \pi + x$ in the interval $(-\pi, \pi)$. 7

Or

Find a Fourier series for the function defined by the equation

$$f(x) = \begin{cases} -1 & \text{for } -\pi < x < 0 \\ 0 & \text{for } x = 0 \\ 1 & \text{for } 0 < x < \pi \end{cases}$$

2. (a) Write when Laplace transform of a function exists. 1
- (b) Write the value of (i) $L\{1\}$ and (ii) $L\{e^{-t}\}$. 1+1=2
- (c) Find (i) $L\{\cos^2 2t\}$ and (ii) $L\{\cosh t\}$. 2+2=4
- (d) Find the following (any two) : 4×2=8
- (i) $L\{(\sin t - \cos t)^2\}$
- (ii) $L\{(1 + te^t)^2\}$
- (iii) $L\{t^2 \cos 2t\}$
- (e) Find the Laplace transform of n th order derivative of a function. 5

Or

If $L\{F(t)\} = f(s)$, then show that

$$L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$$

3. (a) Write the value of $L^{-1}\left\{\frac{1}{s^2+4}\right\}$. 1

(b) Find (i) $L^{-1}\left\{\frac{1}{2s-7}\right\}$ and (ii) $L^{-1}\left\{\frac{9}{s^2+9}\right\}$.
2+2=4

(c) Find $L^{-1}\left\{\frac{e^{-s}}{s^2+1}\right\}$. 2

(d) Find the following (any one) : 3

$$(i) L^{-1}\left\{\frac{s}{(s^2+a^2)(s^2+b^2)}\right\}$$

$$(ii) L^{-1}\left\{\frac{s+1}{s^2+6s+25}\right\}$$

4. (a) Write the Fourier cosine integral formula. 1

(b) If $F[f(x)] = f(p)$, then write the value of $F[f(ax)]$. 1

(c) Find the Fourier cosine transform of the function defined by

$$f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x \geq a \end{cases} \quad 3$$

(d) State and prove shifting property of Fourier transform. 6

(e) Answer the following (any two) : $7 \times 2 = 14$

(i) Find Fourier transform of $f(x)$ defined by $f(x) = e^{-\frac{x^2}{2}}$, $-\infty < x < \infty$.

(ii) Find Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

(iii) Find Fourier transform of $f(x) = xe^{-x}$, $0 \leq x < \infty$.

5. (a) Write the value of $L\left\{\frac{\partial^2 y}{\partial x^2}\right\}$. 1

(b) Find the transformed ordinary differential equation of

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$$

by using Laplace transform. 2

(c) Solve the following using Laplace transform (any two) : $6 \times 2 = 12$

(i) $4 \frac{d^2 y}{dt^2} + \pi^2 y = 0$, $y(0) = 2$, $y'(0) = 0$

(ii) $\frac{d^2 y}{dt^2} + y = 6 \cos 2t$, $y(0) = 3$, $y'(0) = 1$

(iii) $\frac{d^2 y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$, $y(0) = 4$, $y'(0) = -2$

(iv) $\frac{\partial y}{\partial t} = 2 \frac{\partial^2 y}{\partial x^2}$, $y(0, t) = 0 = y(5, t)$,
 $y(x, 0) = 10 \sin 4\pi x$

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