4 SEM TDC PHYH (CBCS) C 8

2023

(May/June)

PHYSICS

(Core)

Paper : C–8

(Mathematical Physics—III)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×4=4

   (a) The value of $|3z_1 - 4z_2|$, if $z_1 = 2 + i$ and $z_2 = 3 - 2i$, is

      (i) 157

      (ii) 167

      (iii) $\sqrt{157}$

      (iv) $\sqrt{167}$

(Turn Over)

(b) The value of the integral $\oint_C \frac{1}{z} dz$, if the point $z = 0$ lies within the curve C, is

(i) $\frac{1}{2\pi i}$

(ii) 0

(iii) $2\pi i$

(iv) None of the above

(c) The Laplace transform $f(s)$ of $F(t) = t$ is

(i) 1

(ii) s

(iii) s^2

(iv) $1/s^2$

(d) If $g(\omega)$ is the Fourier transform of $f(t)$, the Fourier transform of $f(at)$ is

(i) $\frac{1}{a} g\left(\frac{\omega}{a}\right)$

(ii) $\frac{1}{\omega} g\left(\frac{\omega}{a}\right)$

(iii) $\frac{1}{\omega} g\left(\frac{a}{\omega}\right)$

(iv) None of the above

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2. Answer the following : 2×5=10

(a) Express the equation $x^2 + y^2 = 36$ in terms of the conjugate co-ordinates $-5 + 5i$.

(b) Express the complex number $-1 + \sqrt{3}i$ in polar form.

(c) Differentiate between the terms 'essential singularity' and 'removable singularity' with examples.

(d) Using the Fourier integral formula, derive the Fourier cosine transformation.

(e) Prove that if $f(s)$ is the Laplace transform of $F(t)$, the Laplace transform of $e^{at} F(t)$ is $f(s-a)$.

3. Answer the following :

(a) State the Cauchy's residue theorem. Prove that $\int_0^\infty \frac{dx}{x^6 + 1} = \frac{\pi}{3}$. 1+4=5

(b) If $f(z) = u + iv$ is analytic and $u = e^{-x}(x \sin y - y \cos y)$, prove that u is harmonic. Also, find the value of v. 2+3=5

(c) State the Cauchy's integral formula for derivative of a function. Evaluate the integral $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 3$. 1+3=4

(d) Find the residues of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

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(e) Find the Laurent series of $\dfrac{e^{2z}}{(z-1)^3}$

about $z = 1$.

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4. Find the Fourier transform of the following functions (any *two*) : $3 \times 2 = 6$

(i) $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$

(ii) $f(x) = Ne^{-\alpha x^2}$ (N and α are constants)

(iii) $f(x) = \dfrac{1}{2\pi}\left(\dfrac{a^2}{x^2+a^2}\right)$

5. Find the Laplace transform of the following functions (any *two*) : $3 \times 2 = 6$

(i) $f(t) = \dfrac{e^{at}-1}{a}$

(ii) $f(t) = \cos^2 t$

(iii) $f(t) = Kt$, where K is a constant

6. Write short notes on the following (any *two*) : $3 \times 2 = 6$

(a) Singularities of a complex function
(b) Fourier transforms and its applications
(c) Convolution theorem

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